Problems

Example

The Earth is about 150000000km from the Sun. Astronomers call this distance one astronomical unit (1 AU). The distances of other planets from the sun can be expressed in AU's. For example, if a planet were twice as far its distance is 2 AU. In 1776, the astronomer Johann Bode found a sequence that he thought could determine each planet's distance from the sun in Aus.

Planet	**Bode's Distance (AU)**	Actual Distance (AU)
Mercury	$\frac{0+4}{10}=0.4$	0.387
Venus	$\frac{3+4}{10}=0.7$	0.723
Earth	$\frac{6+4}{10}=1.0$	1.0
Mars	$\frac{12+4}{10}=1.6$	1.524
Asteroids	$\frac{24+4}{10}=2.8$	2.3 to 3.3
Jupiter		5.203
Saturn		9.555
Uranus		19.22
Neptune		30.11
Pluto		39.84

a) Continue the sequence for the last five planets
b) Calculate each planets distance using Bode's Law
c) Which planet(s) does Bode's Law fail.

Solution:

Jupiter $\frac{48+4}{10}=5.2$

Saturn $\frac{96+4}{10}=10$

Uranus $\frac{192+4}{10}=19.6$

Neptune $\frac{384+4}{10}=38.8$

Pluto $\frac{768+4}{10}=77.2$

Neptune and Pluto fail.

Example

The time from one full moon to the next is 29.53 days. If the first full moon of a year occurred 12.31 days into the year.

a) How many days into the year did the 9th full moon occurs?
b) At what time of the day did the 9th full moon occur?

Solution:

$$= 12.31 + (9-1)\,29.53$$

$$\text{days} = 248.55$$

$$0.55 \times 25 = 13.2 hrs$$

$$0.2 \times 60 = 12 \min$$

The 9th full moon occurred at 1:12 in the afternoon.

Example

If you had one grain of wheat on the first square of a chess board, two grains on the second square, four on the third, eight on the fourth. How many grains of wheat would you have by the time you finish the 64th square?

Solution:

This is a geometric series

$$a = 1, r = 2, n = 64$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1((2)^{64} - 1)}{2 - 1}$$

$$\doteq 1.8 \times 10^{19}$$

Note: a grain of wheat has a mass of 0.065g, therefore the weight of the wheat would be 1200000000000000 kilograms

Example

If n is chosen from the set of positive integers $\{1, 2, 3, ..., 24, 25\}$, for which values is $n^2 + 2n + 4$ divisible by 7? Can you generalize for n any integer?

Solution:

If $n^2 + 2n + 4$ is divisible by 7, then one of the factors or the terms must also be divisible by 7.

It cannot be factored

Therefore

$$n^2+2n+4=\{7,14,21,28,...\}$$

$$n^2+2n-3=0$$

$$n^2+2n-10=0$$

$$\vdots$$

Once you have a value7 increase of that factor is a value.
1 and 4 are values
Therefore n=1,4,8,11,15,18,22,25,29
Or
1 more than the multiples of 7
4 more than the multiples of 7