Finding Distance between Two Points on the Coordinate Plane

1. GETTING THE IDEA

You can use the Pythagorean theorem to find the distance between two points in the coordinate plane. The diagram below shows how the distance, \( d \), between two points can be represented by the length of the hypotenuse of a right triangle.

Find the horizontal and vertical distances that are the lengths of the legs of the right triangle. Then substitute into the formula \( a^2 + b^2 = c^2 \) to find the distance, \( d \).

horizontal distance from (2, 2) to (6, 5) = \(|2 - 6| = |-4| = 4\)
vertical distance from (2, 2) to (6, 5) = \(|2 - 5| = |-3| = 3\)

\[ a^2 + b^2 = c^2 \]
\[ 4^2 + 3^2 = c^2 \]
\[ 16 + 9 = c^2 \]
\[ 25 = c^2 \]
\[ 5 = c \]

The distance between the points is 5 units.
Example 1
Find the distance between the points (1, 6) and (7, 2). Round your answer to the nearest tenth.

Strategy
Use the Pythagorean theorem.

Step 1
Plot the points, and connect them with a segment. Then draw a right triangle that has the segment as its hypotenuse.

Step 2
Identify $a$, $b$, and $c$.

The length of the hypotenuse is $d$, so let $c = d$.

Use absolute value to find the horizontal and vertical distances that are the lengths of the legs.

horizontal distance from (1, 6) to (7, 2) = $|1 - 7| = |-6| = 6$

vertical distance from (1, 6) to (7, 2) = $|6 - 2| = 4$

The lengths of the legs are 4 units and 6 units. So, $a = 4$ and $b = 6$.

Step 3
Substitute into $a^2 + b^2 = c^2$. Then solve the equation for $d$.

$a^2 + b^2 = c^2$
$4^2 + 6^2 = d^2$
$16 + 36 = d^2$
$52 = d^2$
$\sqrt{52} = d$
$7.2 \approx d$

Solution
The distance between the points (1, 6) and (7, 2) is approximately 7.2 units.
Example 2
Find the distance between the points \((-4, -5)\) and \((3, 0)\). Round your answer to the nearest tenth.

**Strategy**
Use the Pythagorean theorem.

**Step 1**
Plot the points, and connect them with a segment. Then draw a right triangle that has the segment as its hypotenuse.

**Step 2**
Identify \(a\), \(b\), and \(c\).

The length of the hypotenuse is \(d\), so let \(c = d\).

Use absolute value to find the horizontal and vertical distances that are the lengths of the legs.

horizontal distance from \((-4, -5)\) to \((3, 0)\) = \(|-4 - 3| = |-7| = 7\)
vertical distance from \((-4, -5)\) to \((3, 0)\) = \(|-5 - 0| = 5\)

The lengths of the legs are 7 units and 5 units. So, \(a = 7\) and \(b = 5\).

**Step 3**
Substitute into \(a^2 + b^2 = c^2\). Then solve the equation for \(d\).

\[
\begin{align*}
  a^2 + b^2 &= c^2 \\
  7^2 + 5^2 &= d^2 \\
  49 + 25 &= d^2 \\
  74 &= d^2 \\
  \sqrt{74} &= d \\
  8.6 &\approx d
\end{align*}
\]

**Solution**
The distance between the points \((-4, -5)\) and \((3, 0)\) is approximately 8.6 units.
Example 3
Cali drew the map below showing her house and the houses of two of her friends. On the map, each unit square represents 1 square mile. What is the distance between Jenna’s house and Mira’s house to the nearest tenth of a mile?

Strategy
Use the Pythagorean theorem.

Step 1
Connect the points representing Jenna’s house and Mira’s house with a segment. Then draw a right triangle that has the segment as its hypotenuse.

Step 2
Identify $a$, $b$, and $c$.
The length of the hypotenuse is $d$, so let $c = d$.
Use absolute value to find the horizontal and vertical distances that are the lengths of the legs.

horizontal distance from $(-2, 4)$ to $(4, -2) = | -2 - 4 | = | -6 | = 6$
vertical distance from $(-2, 4)$ to $(4, -2) = | 4 - (-2) | = | 4 + 2 | = 6$
The legs are both 6 units long. So, $a = 6$ and $b = 6$.

Step 3
Substitute into $a^2 + b^2 = c^2$. Then solve the equation for $d$.

\[
\begin{align*}
6^2 + 6^2 &= d^2 \\
36 + 36 &= d^2 \\
72 &= d^2 \\
\sqrt{72} &= d \\
8.5 &\approx d
\end{align*}
\]

Solution
The distance between Jenna’s house and Mira’s house is approximately 8.5 miles.
Example 4
Find the perimeter of the trapezoid.

**Strategy**
Find the length of each side, using the Pythagorean theorem where necessary. Then find the sum of the lengths of the sides.

**Step 1**
Find the lengths of the horizontal and vertical sides.

- Use absolute value to find the horizontal and vertical distances that are the lengths of sides \( AB, BC, \) and \( AD \).
- \( AB = |1 - 4| = |-3| = 3 \) units
- \( BC = |1 - 5| = |-4| = 4 \) units
- \( AD = |1 - 7| = |-6| = 6 \) units

**Step 2**
Use the Pythagorean theorem to find the length of the slanted side.

To find the length of side \( CD \), draw a right triangle that has \( CD \) as its hypotenuse.

Identify \( a, b, \) and \( c \).

- The length of the hypotenuse is \( x \), so let \( c = x \).
- The vertical distance is equal to \( AB \), so is 3 units.

Use absolute value to find the horizontal distance.

- Horizontal distance from \((5, 1)\) to \((7, 1)\) = \(|5 - 7| = |-2| = 2\)

The lengths of the legs are 3 units and 2 units. So, \( a = 3 \) and \( b = 2 \).

\[
a^2 + b^2 = c^2
\]
\[
3^2 + 2^2 = x^2
\]
\[
9 + 4 = x^2
\]
\[
13 = x^2
\]
\[
\sqrt{13} = x
\]
\[
3.6 \approx x
\]

So, the length of side \( CD \) is approximately 3.6 units.
2 COACHED EXAMPLE

Find the approximate length of the longer diagonal of the kite \(ABCD\) to the nearest tenth of a unit.

Draw the diagonals of the kite, \(\overline{AC}\) and \(\overline{BD}\). Which is the longer diagonal? _____

Draw a right triangle using the longer diagonal as its hypotenuse. Label the diagonal “\(x\)”.

Look at the right triangle you drew. Identify the lengths of \(a\), \(b\), and \(c\).

\[
\begin{align*}
  a &= \text{_____ units} \\
  b &= \text{_____ units} \\
  c &= \text{_____ units} 
\end{align*}
\]

Write and solve an equation to find the value of \(x\).
Round your answer to the nearest tenth.

\[
\begin{align*}
  a^2 + b^2 &= c^2 \\
  \text{_____}^2 + \text{_____}^2 &= x^2 \\
  \text{_____} + \text{_____} &= x^2 \\
  \text{_____} &= x^2 \\
  \sqrt{\text{_____}} &= x \\
  \text{_____} &= x
\end{align*}
\]

The length of the longer diagonal is approximately _____ units.
1. Find the perimeter of rhombus $ABCD$ below.

The perimeter is ________ units.

2. A trapezoid has vertices $(-6, 2), (6, 7), (6, -4),$ and $(-6, -4)$.

   **Part A**
   
   Graph the trapezoid on the grid below.

   **Part B**
   
   What is the perimeter of the trapezoid? Show your work.
The map below shows Tyler’s house and the houses of two of his friends, Pedro and Jake. On the map, each unit square represents 1 square mile. Select True or False for each statement.

A. Tyler lives closer to Jake than to Pedro.  ○ True  ○ False

B. Pedro lives closer to Jake than to Tyler.  ○ True  ○ False

C. Jake lives about 0.4 mile closer to Tyler than to Pedro.  ○ True  ○ False

D. The sum of the distances between the boys’ houses is less than 18 miles.  ○ True  ○ False

E. The sum of the distances between the boys’ houses is greater than 20 miles.  ○ True  ○ False

A kite has vertices (−6, −3), (−5, 2), (6, 5), and (−1, −4). What is the difference between the lengths of the two diagonals of the kite? Round your answer to the nearest hundredth of a unit. Show your work.
5. Use expressions from the box to complete the table.

<table>
<thead>
<tr>
<th>Points</th>
<th>Distance between the Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−3, 5) and (6, −1)</td>
<td>(\sqrt{9^2 + 4^2})</td>
</tr>
<tr>
<td>(2, −3) and (−2, −6)</td>
<td>(\sqrt{8^2 + 7^2})</td>
</tr>
<tr>
<td>(7, −2) and (3, −1)</td>
<td>(\sqrt{4^2 + 3^2})</td>
</tr>
<tr>
<td>(−2, −4) and (4, −1)</td>
<td>(\sqrt{4^2 + 1^2})</td>
</tr>
</tbody>
</table>

6. A coordinate grid is superimposed on a map of the county park. The grid shows a rectangular playground with vertices (1, 4), (5.5, 7), (3, 1), and (7.5, 4). Each unit on the grid represents 10 feet.

**Part A**
Graph the rectangle that represents the playground on the grid below.

**Part B**
What is the area of the playground? Round your answer to the nearest square foot. Show your work.
Circle the numbers that make the sentence true

The distance between the points \((4, \frac{3}{5})\) and \(( -7, 5)\) is 13 units.

A plan for a mosaic made from tiles calls for triangles of different sizes and shapes. Adrianna is drawing patterns for the different shapes on grid paper. The figure below shows one side of what will be a tile in the shape of an equilateral triangle.

If the third vertex of the triangle is located in the first quadrant, what are the coordinates of the vertex? Round the coordinates to the nearest hundredth. Use words, numbers, and diagrams to justify your answer.