**Fluid Dynamics**

Practice Questions

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**Question 1**

A person sips a drink through a straw. At which of the following three positions is the pressure the lowest?

I) Inside the person's mouth
II) At the surface of the drink
III) At the bottom of the drink

A) Only at position I  
B) Only at position II  
C) Only at position III  
D) Both at position I and III  
E) Both at position I and II

The fluid is pushed into the mouth by the atmospheric pressure. Because the surface of the fluid is open to the atmosphere, the surface is at atmospheric pressure, and the pressure in the mouth must be lower than the atmospheric.

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**Question 2**

The circulatory system can be modeled as an interconnected network of flexible pipes (the arteries and veins) through a pump (the heart) causes blood to flow. Which of the following actions, while keeping all other aspects of the system the same, would NOT cause the velocity of the blood to increase inside a vein?

A) Expanding the vein's diameter  
B) Cutting off blood flow to some other area of the body  
C) Increase the heart rate  
D) Increasing the total amount of blood in the system  
E) Increasing the pressure difference of the vein

Flow rate (volume of flow per second) is the area of the pipe times the speed of the flow. If the flow rate is constant and you increase the diameter (thus the area) of the vein, then the velocity must decrease.

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**Question 3**

A pirate ship hides out in a small inshore lake. It carries twenty ill-gotten treasure chests in its hold. But lo, on the horizon, the lookout spies a gunboat. To get away, the pirate captain orders the heavy treasure chests jettisoned. The chests sink to the lake bottom. What happens to the water level of the lake?

A) The water level rises  
B) The water level drops  
C) The water level does not change

When the treasure is in the hold, it is floating on the water. So by Archimedes' principle, the treasure must displace a volume of water equal to the weight of the treasure. However, when the treasure is resting on the bottom of the lake, the treasure does not have to be supported by the buoyancy force. Thus the treasure only displaces a volume of water equal to its own volume (The density of the treasure is greater than that of water, it sinks when floating but when sunk).
**Question 4**

Brian saves 2-litre soda bottles so that he can construct a raft and float out onto a pond. If Brian has a mass of 80 kg, what minimum number of bottles is necessary to support him? The density of the water is 1000 kg/m$^3$, and 1000 L = 1 m$^3$

A) 1600 bottles
B) 800 bottles
C) 200 bottles
D) 40 bottles
E) 4 bottles

Since Brian is floating in equilibrium, his weight must equal the buoyancy force on him.

\[
F = \rho \cdot V \cdot g
\]

\[
F = 800 \text{N}
\]

\[
V = \frac{81.6}{0.0816} = 40 \text{ bottles}
\]

**Question 5**

A hydraulic lift is designed for a gain of 100, so that a 10 N force applied at the input piston will produce a 1000 N at the output piston. If the radius of the input piston is 2 cm, the radius of the output piston is:

A) 200 cm
B) 0.02 cm
C) 400 cm
D) 20 cm
E) 0.05 cm

We need only apply Pascal’s Principle keeping in units.

\[
\frac{F_\text{out}}{F_\text{in}} = \frac{R_\text{out}}{R_\text{in}}
\]

\[
\frac{1000 \text{N}}{10 \text{N}} = \frac{R_\text{out}}{2 \text{ cm}}
\]

\[
R_\text{out} = 20 \text{ cm}
\]

**Question 6**

A cube of side L is made of a substance that is ¼ as dense as water. When placed in a calm water bath, the cube will:

A) Float with ½ L above the surface
B) Sink to the bottom
C) Float with ¼ L above the surface
D) Float with ¾ L above the surface
E) Float with a submerged object below the surface

We need only apply Archimedes’s Principle keeping in mind units. We know that since the density is less than water, it will float.

\[
\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{submerged}}}{\rho_{\text{water}}}
\]

\[
\frac{1}{4} = \frac{\frac{1}{4} \cdot L^3}{L^3}
\]

**Question 7**

A cylindrical pipe has a radius of 12 cm in one region where the fluid speed is 0.2 m/s. In another region, the pipe is narrower with a radius of 4 cm. The fluid speed in this region is most nearly:

A) 9 m/s
B) 0.022 m/s
C) 1.8 m/s
D) 0.011 m/s
E) 0.2 m/s

This is an application of the Continuity Equation. Ensure that you are using the correct units.

\[
\frac{A_1 \cdot v_1}{A_2 \cdot v_2} = \frac{r_1^2 \cdot v_1}{r_2^2 \cdot v_2}
\]

\[
v_2 = \frac{(0.12 \text{ m})^2 \cdot (0.2 \text{ m/s})}{(0.04 \text{ m})^2}
\]

\[
v_2 = 1.8 \text{ m/s}
\]
Question 8

A water pump is attached to the left end of a horizontal pipe that consists of a rigid section and a flexible second section that can have its cross-sectional area adjusted. A pool needs to be filled with the output of the flexible section. Which of the following will increase the rate at which the pool will fill?

I. Increase the pump pressure
II. Decrease the cross-sectional area of the second section
III. Increase the cross-sectional area of the second section

A) I only
B) II only
C) III only
D) I and II only
E) I and III only

First Bernoulli’s Equation tells us that by increasing the pressure at the input will result with an increase of the velocity at the output for any cross section. The continuity equation tells us that changing the cross-sectional area will not affect the amount that flows into the pool.

Question 9

An ideal fluid flows through a pipe that runs up an incline and gradually rises to a height H. The cross sectional area of the pipe is uniform. Compared with the flow at the bottom of the incline, the flow at the top is:

A) Moving slower at lower pressure
B) Moving slower at higher pressure
C) Moving at the same speed at lower pressure
D) Moving at the same rate at higher pressure
E) Moving faster at lower pressure

Since the area did not change, the continuity equation implies that the fluid velocity is the same. Bernoulli’s Equation tells us that the pressure at height H must be less than:

\[ P_{\text{top}} = P_{\text{bottom}} - \rho g H \]

Question 10

A beaker of water sits on an electric scale with an initial reading of 30 N. A mass with 3 times the density of water hangs from a spring scale with an initial reading of 6 N. Still attached to the spring scale, the mass is completely immersed in the water. The reading on the two scales (in electric, spring order) will be:

A) 30 N, 2 N
B) 32 N, 6 N
C) 36 N, 4 N
D) 32 N, 4 N
E) 36 N, 4 N

Since the density is 3 times as great as water, the object will experience a buoyant force of one-third its weight, or 2 N. The spring scale will then read 6 + 2 = N. The reaction to the buoyant force acts on the water and eventually the electronic scale, producing an extra downward force of 2 N. The scale then reads 32 N.

Question 11

The water tower in the drawing is drained by a pipe that extends to the ground. The amount of water in the top of the spherical portion of the tank is significantly greater than the amount of water in the supporting column (density of water 1000 kg/m³):

A) What is the absolute pressure at the position of the valve if the valve is closed, assuming that the top surface of the water at point P is at atmospheric pressure (10⁵ N/m²)?
B) Now the valve is opened. Thus, the pressure at the valve is forced to be atmospheric pressure. What is the speed of the water past the valve?
C) Assuming that the radius of the circular valve opening is 10 cm, find the volume flow rate out of the valve.
D) Considering that virtually all of the water is originally contained in the top spherical portion of the tank, estimate the initial volume of the water contained by the water tower.
E) Estimate how long it would take to drain the tank completely using this single valve.
What is the absolute pressure at the position of the valve if the valve is closed?

Now the valve is opened; thus, the pressure at the valve is forced to be the same as the pressure at the point P.

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Estimate how long it would take to drain the tank completely using this single valve.

Flow rate \( A \cdot v \)

\[
\text{Flow rate} = \pi r^2 v
\]

\[
= \pi \left( 0.1m \right)^2 \left( \frac{17m}{s} \right)
\]

\[
= 0.314m^2 \cdot \frac{17m}{s}
\]

\[
= 0.53 m^3/s
\]

The amount of water in the top of the spherical portion of the tank is significantly greater than the amount of water in the supporting column (density of water 1000 kg/m³).

A) What is the absolute pressure at the position of the valve if the valve is closed, assuming that the top surface of the water at point P is at atmospheric pressure (10^-5 N/m²)?
B) Now the valve is opened; thus, the pressure at the valve is forced to be atmospheric pressure. What is the speed of the water past the valve?
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Question 12

A piston of cross section $A_x$ can move inside a long tube that's connected to a large cylindrical reservoir with cross section $A_y$ of fluid that has a density of $\rho$. Currently a piston of mass $M$ is supported at the top of the cylinder at a height $H$ above the long tube. Compressed air is pumped to the left of the small piston and maintains it in its current position.

A) Find the pressure of the compressed air?

B) The piston needs to be raised an amount $\Delta y$.
   i. How far must the small piston move?
   ii. How much must the air pressure be increased to lift the piston?

Without the upper piston, the pressure at the lower piston is just the fluid pressure at depth $H$. Adding the piston creates an extra pressure which will be transmitted, undiminished, to all points within the fluid. The total pressure the compressed air must supply is:

$$P_{\text{air}} = P_{\text{fluid}} + \rho g H + \frac{Mg A}{A_x}$$
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B) The piston needs to be raised an amount $\Delta y$.
   i. How far must the small piston move?
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The increase in pressure is needed to support the extra fluid in $\Delta y$. The old pressure could already support the piston and the fluid to height $H$, so

$$\Delta P = \rho g \Delta y$$

Question 13

A vendor at a flea market for the rich and famous claims the crown he is selling is pure gold. On a precise spring scale, you weigh the crown and read a value of 25.14 N. Next, you immerse the crown in water while it is still hanging from the scale, this time getting a reading of 20.65 N. since you know the ratio of gold density to water density is 19.32, what do you conclude from the vendor’s claim?

The difference in the two scale readings is the buoyant force. Since this is the weight of the displaced water, you have:

$$W_{water} = \rho_{water} \cdot V_{water} \cdot g$$

With $M = 25.14$ N and $W_{water} = 20.65$ N, you can find $\rho_{water}$:

$$\rho_{water} = \frac{W_{water}}{M \cdot g} = \frac{20.65}{25.14 \cdot g}$$

Since $\rho_{water}$ is less than 5.60, the crown is much less dense than pure gold, the vendor is mistaken.
A large storage container in a commercial wine cellar is cylindrical in shape. To test the contents (density of 1000 kg/m\(^3\)), you can insert a tapping mechanism near the base of the cylinder. The mechanism consists of a larger cylindrical pipe of radius 0.5 cm that narrows to 0.2 cm at the spigot. Currently, the tapping device is 2 m below the wine level in the container. Assume the space above the wine in the container is maintained at atmospheric pressure and that wine is an ideal fluid. You may also assume that loss of wine through the spigot does not appreciably change the volume of wine in the container.

a) Find the time it will take to fill a 1 L flask at the spigot.
b) Determine the speed of the fluid as it enters the tapping device.
c) Find the difference between atmospheric pressure and the fluid pressure just inside the tapping device.

We can apply the continuity equation

\[
A \cdot V = A_{in} V_{in} + A_{out} V_{out}
\]

We need the output velocity at the spigot

\[
V_{out} = \sqrt{\frac{2 \left( \rho_{in} g h + p_{atm} \right) - \rho_{out} g h - p_{atm}}{\rho_{out}}} = \sqrt{\frac{2 \left( \rho_{in} g h + p_{atm} \right)}{\rho_{out}}}
\]

\[
V_{out} = \sqrt{\frac{2 \left( 1000 \text{ kg/m}^3 \cdot 9.81 \text{ m/s}^2 \cdot 2 \text{ m} + 1 \text{ atm} \cdot 101325 \text{ Pa} \right)}{1000 \text{ kg/m}^3}} = 6.32 \text{ m/s}
\]

We also assume that loss of wine through the spigot does not appreciably change the volume of wine in the container.

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Question 14

a) Find the time it will take to fill a 1 L flask at the spigot.
b) Determine the speed of the fluid as it enters the tapping device.

We can apply the Bernoulli’s Equation:

\[ p_0 + \rho g h + p_a = \frac{1}{2} \rho v^2 + p_a + \frac{1}{2} \rho v^2 + \rho g h + p_a \]

\[ p_a = p_a + \rho g h + \frac{1}{2} \rho v^2 + \frac{1}{2} \rho v^2 + \rho g h + p_a \]

\[ p_a = \rho g D + \frac{1}{2} \rho v^2 + \frac{1}{2} \rho v^2 + \rho g h + p_a \]

\[ \Delta p = \left[ 1000 \text{ kg/m}^3 \left( 9.8 \text{ m/s}^2 \right) \right] \left( 2 \text{ m} \right) - \frac{1}{2} \left( 1000 \text{ kg/m}^3 \right) \left( 1.01 \text{ m} \right)^2 \]

\[ = 1.91 \times 10^6 \text{ Pa} \]

Question 15

The figure below shows a tank open to the atmosphere and filled to depth \( D \) with a liquid of density \( \rho \). Suspended from a string is a block of density \( \rho_B \) (which is greater than \( \rho \)), whose dimensions are \( x \), \( y \), and \( z \) (metres). The top of the block is at depth \( h \) metres below the surface of the liquid. We shall use hydrostatic pressure and pressure formulas:

\[ P_{asw} = p_a + \rho g h \]

\[ P_{bottom} = p_a + \rho g \left( h + \frac{1}{2} z \right) \]

\[ F_{top} = P_{top} A = (p_a + \rho g h) x y \]

\[ F_{bottom} = P_{bottom} A = (p_a + \rho g \left( h + \frac{1}{2} z \right)) x y \]

\[ \Delta P = \left( \frac{\rho - \rho_B}{\rho_B} \right) \rho g h \]

We note that the four sides are at an average depth of \( h + \frac{1}{2} z \):

\[ P_{sides} = p_a + \rho g \left( h + \frac{1}{2} z \right) \]

\[ F_{sides} = P_{sides} A = \left( p_a + \rho g \left( h + \frac{1}{2} z \right) \right) x y \]

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The figure below shows a tank open to the atmosphere and filled to depth \( D \) with a liquid of density \( \rho_1 \). Suspended from a string is a block of density \( \rho_2 \) (which is greater than \( \rho_1 \)), whose dimensions are \( x \), \( y \), and \( z \) (metres). The top of the block is at depth \( h \) metres below the surface of the liquid.

(a) Find the force due to the pressure on the top surface of the block and on the bottom surface.
(b) What are the average forces due to the pressure on the other four sides of the block? Sketch these forces.
(c) What is the total force on the block due to the pressure?
(d) Find an expression for the buoyant force on the block. How does your answer here compare to your answer in part c)
(e) What is the tension in the string?

By Archimedes' Principle, the buoyant force on the block is upward with magnitude

\[
F_{\text{buoy}} = \rho_1 V g
\]

This is the same as the answer in c)

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Since the four forces in b) add up to zero, and the Bottom force is greater than the Top Force

\[
F_{\text{out}} = F_{\text{in}} = F_g
\]

\[
F_{\text{buoy}} = (p_{\text{out}} + \rho_2 g h) xy
\]

\[
F_{\text{buoy}} = (p_{\text{in}} + \rho_2 g (h + z)) xy
\]

\[
F_{\text{out}} = (p_{\text{out}} + \rho_2 g (h + z)) xy - (p_{\text{in}} + \rho_2 g h) xy = \rho_2 g xyz
\]

This is the same as the answer in c)

Question 15

The figure below shows a large cylindrical tank of water, open to the atmosphere, filled with water to depth \( D \). The radius of the tank is \( R \). At a depth \( h \) below the surface, a small hole of radius \( r \) is punctured in the side of the tank, and the point where the emerging stream strikes the level ground is labelled X. In parts (a) through (c), assume that the speed with which the water level in the tank drops is negligible.

(a) At what speed does the water emerge from the hole?
(b) How far is point X from the edge of the tank?
(c) Assume that a second small hole is punctured in the side of the tank, a distance of \( h/2 \) directly above the hole shown in the figure. If the stream of water emerging from the second hole also lands at Point X, find \( h \) in terms of \( D \).
(d) For this part, do not assume that the speed with which the water level in the tank drops is negligible, and derive an expression for the speed of efflux from the hole punctured at depth \( h \) below the surface of the water. Write your answer in terms of \( r, R, h, \) and \( g \).
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a) At what speed does the water emerge from the hole? 

b) How far is point $X$ from the edge of the tank? 

c) Assume that a second small hole is punctured in the side of the tank, a distance of $h/2$ directly above the hole shown in the figure. If the stream of water emerging from the second hole also lands at Point $X$, find $h$ in terms of $D$. 

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\[ \frac{1}{2} \rho v^2 + \rho gh + p_1 = \frac{1}{2} \rho v'^2 + \rho gh + p_2 \]

Since open to the air, 

\[ 0 = \rho gh'_{	ext{top}} + p_{	ext{atm}} = \frac{1}{2} \rho v'^2 + \rho gh'_{	ext{atm}} + p_{	ext{atm}} \]

Applying Bernoulli's Theorem (it contains velocity terms that are independent to each other on each side of the equation).

\[ \rho g h'_{	ext{atm}} = \frac{1}{2} \rho v'^2 = \frac{1}{2} \rho v^2 \]

**Question 16**

The figure below shows a large cylindrical tank of water, open to the atmosphere, filled with water to depth $D$. The radius of the tank is $R$. At a depth $h$ below the surface, a small hole of radius $r$ is punctured in the side of the tank, and the point where the emerging stream strikes the level ground is labelled $X$, find $h$ in terms of $D$. 

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**Question 16**
1. Let's determine the velocity first.

We shall apply Bernoulli's equation to Point 1 and the exit point.

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \]

We will define the ground (y=0) to be zero, therefore this term goes to zero.

\[ P_1 - P_\text{atm} = \left( \frac{1000 \text{ kg/m}^3}{2} \right) \left( 9.8 \text{ m/s}^2 \right) \left( 0.5 \text{ m/s} \right) \]

\[ v_1 = \sqrt{10^5 \text{ Pa}} \]

2. In a town’s water system, pressure gauges in still water at street level read 150 kPa. If a pipeline connected to the system breaks and shoots water straight up, how high above the street does the water shoot?

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\[ P_1 = 150 \text{ kPa} \]

\[ v = 17 \text{ m/s} \]

\[ \left( \frac{10^5 \text{ Pa}}{2} \right) = \left( 17 \text{ m/s} \right)^2 \left( 2 \times 9.8 \text{ m/s}^2 \right) (\Delta y) \]

\[ \Delta y = 15 \text{ m} \]

3. Water circulates throughout the house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm radius pipe in the basement under a pressure of 9.0 atm, what will be the flow speed and pressure in a 2.5 cm radius pipe on the second floor 5.0 m above?

Recall: 1 atm = 1 x 10^5 Pa

We want both velocity and pressure. Continuity equation is easiest for velocity, and Bernoulli can be used for pressure.

\[ \frac{A_1 v_1}{A_2 v_2} = \frac{P_1 - P_\text{atm}}{P_2 - P_\text{atm}} \]

\[ \pi (0.04\text{m})^2 \left( 0.50\text{m/s} \right) = \pi (0.025\text{m})^2 v_2 \]

Now, let's determine the pressure

\[ P_\text{atm} + P \rho \left( \frac{1}{2} \rho v_1^2 \right) = P_\text{atm} + P \rho \left( \frac{1}{2} \rho v_2^2 \right) \]

\[ 9.0 \times 10^5 \text{ Pa} + \left( 0.50 \text{ m/s} \right)^2 \left( 1000 \text{ kg/m}^3 \right) = P_\text{atm} + \left( 0.25 \text{ m/s} \right)^2 \left( 1000 \text{ kg/m}^3 \right) \]

\[ P_\text{atm} = 2.5 \times 10^5 \text{ Pa} \]

\[ = 2.5 \text{ atm} \]
Question 21

Water travels through a 9.6 cm radius fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose radius is 2.5 cm. What is the speed of the water coming out of the nozzle? Suppose the pressure in the fire hose is 350 kPa. What is the pressure in the nozzle?

\[ A v_1 = A v_2 \]
\[ \pi (0.096 m)^2 v_1 = \pi (0.025 m)^2 v_2 \]
\[ v_2 = 19 m/s \]

\[ P_{\text{inv}} + \frac{1}{2} \rho v_1^2 = P_{\text{out}} + \frac{1}{2} \rho v_2^2 \]
\[ P_{\text{inv}} = P_{\text{out}} + \frac{1}{2} \rho (v_1^2 - v_2^2) \]
\[ P_{\text{out}} = 1.39 \times 10^5 Pa \]
\[ = 1.39 \text{ atm} \]

Question 22

Water flows with constant speed through a garden hose that goes up a step 20.0 cm high. If the water pressure is 143 kPa at the bottom of the step, what is its pressure at the top of the step?

\[ P_B \Rightarrow \frac{P_T}{P_B} = \frac{h_B}{h_T} \]
\[ P_T = 1.41 \times 10^5 Pa \]
\[ = 1.4 \text{ atm} \]

Question 23

Repeat the previous example with the following additional information: (a) the cross-sectional area of the hose at the top of the step is half that at the bottom of the step, and (b) the speed of the water at the bottom of the step is 1.20 m/s.

\[ P_B + \frac{1}{2} \rho v_B^2 = P_T + \frac{1}{2} \rho v_T^2 \]
\[ P_T = P_B + \frac{1}{2} \rho (v_B^2 - v_T^2) \]
\[ P_B = 1.39 \times 10^5 Pa \]
\[ = 1.39 \text{ atm} \]

From continuity equation: \( v_T = 2(1.20 m/s) = 2.40 m/s \)

How fast is the water moving to the right?

Time to hit bottom can:
\[ t = \frac{1.72 m}{9.8 m/s^2} = 0.319 s \]
\[ d = v_T t = (1.72 m/s)(0.319 s) = 0.549 m \]

Question 24

In designing a backyard water fountain, a gardener wants a stream of water to exit from the bottom of one can and land in a second one, as shown. The top of the second can is 0.500 m below the hole in the first can, which has water in it to a depth of 0.150 m. How far to the right of the first can must the second one be placed to catch the stream of water?

\[ v = \sqrt{\frac{2gh}{2}} \]
\[ v = \sqrt{\left(\frac{9.8 m/s^2}{2}\right)(0.150 m)} \]
\[ v = 1.72 m/s \]
Water flows from left to right through the five sections (A, B, C, D, E) of the pipe shown in the drawing, in which section does the water speed increase, decrease, and remain constant? Treat water as an incompressible fluid. (The answer is given at the end of the book.)

<table>
<thead>
<tr>
<th>Speed Increase</th>
<th>Speed Decrease</th>
<th>Speed Is Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A, B</td>
<td>D, E</td>
<td>C</td>
</tr>
<tr>
<td>b. D, E</td>
<td>A, C, E</td>
<td>C</td>
</tr>
<tr>
<td>c. C, D</td>
<td>A, B</td>
<td>E</td>
</tr>
<tr>
<td>d. A, B</td>
<td>C, D</td>
<td>E</td>
</tr>
</tbody>
</table>

Background: The equation of continuity holds the key here. The fact that water can be treated as an incompressible fluid is important.

Fluid is flowing from left to right through a pipe (see the drawing). Points A and B are at the same elevation, but the cross-sectional areas of the pipe are different. Points B and C are at different elevations, but the cross-sectional areas are the same. Rank the pressures at the three points, highest to lowest. (The answer is given at the end of the book.)

a. A and B (tie), C
b. C, A and B (tie)
c. B, C, A
d. C, B, A

Background: Bernoulli’s equation plays the principal role. The equation of continuity is also pertinent.