An object 6 cm tall is placed 40 cm from a thin converging (double convex) lens of $f = 8$ cm. A second converging lens of 12 cm focal length is placed 20 cm from the first lens. Determine the position, size and character of the final image.

The image of the first lens can be used as the object of the second lens.

Since the magnification is negative, the image is inverted at $(0.25)(6\,\text{cm}) = 1.5$ cm tall.
An object 6 cm tall is placed 40 cm from a thin converging (double convex) lens of \( f = 8 \) cm. A second converging lens of 12 cm focal length is placed 20 cm from the first lens. Determine the position, size and character of the final image.

The image of the first lens can be used as the object of the second lens.

Since the magnification is positive, the image is upright.
Multiple Lenses

An object is placed 20 cm from a thin diverging (double concave) lens of \( f = 10 \) cm. A second lens (converging) of 20 cm focal length is placed 10 cm to the right of the first lens. Determine the position, size and character of the final image.

Objects of the second lens can be virtual. Let's move the second lens closer to the first lens (in fact, to its focus):

The object for the second lens is VIRTUAL. Therefore we will use the BST (Burns Schlueter Theorem) for ray tracing. The lens will pretend to have the negative of its focal length and thus opposite properties. The diverging lens will now pretend to be a converging lens.

Note the negative object distance for the 2nd lens.
Problem 1

- Suppose we interchange the converging and diverging lenses in the preceding case.

- What is the relation of the new magnification $m'$ to the original magnification $m$?

  - (a) $m' < m$
  - (b) $m' = m$
  - (c) $m' > m$

---

**Slide 11**

- Since the formula for the magnification is equal to the product of the magnifications of each lens ($m = m_1 \cdot m_2$), you might think that interchanging the lenses does not change the overall magnification.

- This argument misses the point that the magnification of a lens is not a property of the lens, but depends also on the object distance!

- Consider the ray shown which illustrates that the magnification must be $< 1$!

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**Slide 12**

- What is the nature of the final image?

  - (a) real
  - (b) virtual

- The ray used in part A actually shows that the image is real and inverted.

- The equations:
  
  $\frac{1}{s_1} = \frac{1}{f_1} - \frac{1}{x_1}$
  
  $\frac{1}{s_2} = \frac{1}{f_2} - \frac{1}{x_2}$
  
  $\frac{1}{s_1} \cdot \frac{1}{s_2} = \frac{1}{f_1} \cdot \frac{1}{f_2} = \frac{11}{15} \cdot \frac{12}{23}$
The Microscope

Microscope Question

A microscope has an objective of focal length 0.300 cm and a eyepiece focal length of 2.00 cm.

a) Where must the image formed by the objective be for the eyepiece to produce a virtual image 25.0 cm in front of the eyepiece?
b) If the lenses are 20.0 cm apart, what is the distance of the objective from the object on the slide?
c) What is the total magnification of the microscope?
d) What distance would the object have to be from a single lens that gave the same magnification? What would its focal length have to be?
Microscope Question

A microscope has an objective of focal length 0.300 cm and a eyepiece focal length of 2.00 cm.

a) Where must the image formed by the objective be for the eyepiece to produce a virtual image 25.0 cm in front of the eyepiece?

\[ \frac{1}{s} = \frac{1}{s_o} + \frac{1}{s_i} \]

\[ s_i = 20 \text{ cm} \]

\[ s_o = 20 \text{ cm} - s_i \]

\[ = 20 \text{ cm} - 2.17 \text{ cm} \]

\[ = 17.8 \text{ cm} \]

\[ s = \frac{17.8 \text{ cm}}{0.305 \text{ cm}} \]

\[ \approx 58.3 \text{ cm} \]

b) If the lenses are 20.0 cm apart, what is the distance of the objective from the object on the slide?

\[ \frac{1}{s} = \frac{1}{s_o} + \frac{1}{s_i} \]

\[ s_i = 0.3 \text{ cm} \]

\[ s_o = 17.8 \text{ cm} - 0.3 \text{ cm} \]

\[ = 17.5 \text{ cm} \]

\[ s = \frac{17.5 \text{ cm}}{(0.3 \text{ cm})(17.8 \text{ cm})} \]

\[ \approx 0.305 \text{ cm} \]

c) What is the total magnification of the microscope?

\[ M = m_o m_e \]

\[ = \frac{L_e}{f_o} \times \frac{L_o}{f_e} \]

\[ M = \frac{25 \text{ cm}}{0.3 \text{ cm}} \times \frac{20 \text{ cm}}{788.65 \text{ cm}} \]

\[ \approx 833.33 \]

d) What distance would the object have to be from a single lens that gave the same magnification? What would its focal length have to be?

\[ \frac{1}{s} = \frac{1}{s_o} + \frac{1}{s_i} \]

\[ s_i = 0.031 \text{ cm} \]

\[ s_o = 25 \text{ cm} - s_i \]

\[ = 25 \text{ cm} - 0.031 \text{ cm} \]

\[ = 24.969 \text{ cm} \]

\[ f = \frac{0.031 \text{ cm}}{24.969 \text{ cm}} \]

\[ \approx 0.0012 \text{ cm} \]
**The Telescope**

\[ M_{\text{angular}} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} \]

---

**Telescope Question**

An astronomical telescope has an objective of 50 cm focal length. The eyepiece has a focal length of 3.5 cm. How far must these lenses be separated when viewing and object 200 cm from the objective?

Therefore the eyepiece must be placed so that the principal focus is at the location of the objective’s image, to form a virtual image at infinity. Thus, the separation of the two lenses will be:

\[ 66.67 \text{ cm} + 3.5 \text{ cm} = 70.17 \text{ cm} \]

---

**Amazing Eye**

- One of first organs to develop.
- 100 million Receptors
- 200,000 /mm²
- Sensitive to single photons!
- Candle from 12 miles
The Physics of Focusing the Eye

Cornea: $n = 1.38$
Lens: $n = 1.4$
Vitreous: $n = 1.33$

Which part of the eye does most of the light bending?
1) Lens  2) Cornea  3) Retina  4) Cones

Lens and cornea have similar shape and index of refraction. Cornea has air/cornea interface $1.38/1$, 70% of bending. Lens has Lens/Vitreous interface $1.4/1.33$. Lens is important because it can change shape.

Eye (Relaxed)

Determine the focal length of your eye when looking at an object far away.

Object is far away: $s_o = \infty$

Image at retina: $s_i = 25 \text{ mm}$

$f_{\text{relaxed}} = 25 \text{ mm}$

Eye (Tensed)

Determine the focal length of your eye when looking at an object up close (25 cm).

Object is up close: $s_o = 25 \text{ cm} = 250 \text{ mm}$

Want image at retina: $s_i = 25 \text{ mm}$

Recall: $f_{\text{relaxed}} = 25 \text{ mm}$
Near Point, Far Point

- Eye’s lens changes shape (changes $f$)
  - Object at any $d_o$ can have image be at retina ($d_i = \text{approx. 25 mm}$)
- Can only change shape so much
- “Near Point”
  - Closest $d_o$ where image can be at retina
  - Normally, ~25 cm (if far-sighted then further)
- “Far Point”
  - Furthest $d_o$ where image can be at retina
  - Normally, infinity (if near-sighted then closer)

If you are nearsighted...

(far point is too close)

Want to have (virtual) image of distant object, $d_o = \infty$, at the far point, $d_i = -d_{far}$.

Contact lenses form virtual image at far point – becomes object for eye.

Refractive Power of Lens

Diopter $= \frac{1}{f} = \text{POWER}$

where $f$ is focal length of lens in meters.

Person with far point of 5 meters, would need contacts with focal length ~5 meters.

Doctor’s prescription reads:

$\frac{1}{(-5m)} = -0.20 \text{ Diopters}$
If you are farsighted...
(near point is too far)

When object is at \(d_o\), lens must create an (virtual) image at \(-d_{near}\).

Want the near point to be at \(d_o\).

Too close for farsighted eye to focus

\[
\frac{1}{d_o} + \frac{1}{d_{near}} = \frac{1}{f}
\]

Far-sighted eye can focus on this!

Contacts form virtual image at near point – becomes object for eye.

\[
\frac{1}{25cm} + \frac{1}{50cm} = \frac{1}{f_{cm}}
\]

\(f = 50cm\)

\[
\frac{1}{2.5 cm} + \frac{1}{2.3 cm} = \frac{1}{f}
\]

\(f = 2.3 cm\)

The Eye

- **Near Point** is closest distance that can be focused on to the retina
- **Far Point** is distance that relaxed eye can focus onto retina = \(\infty\)

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_{near}} = \frac{1}{2.5 cm}
\]

\(f = 2.5 cm\)

Therefore the normal eye acts as a lens with a focal length which can vary from 2.5 cm (the eye diameter) to 2.3 cm which allows objects from 25 cm to infinity to be focused on the retina.

This is called “accommodation”.

Diopter: \(1/f\) where \(f\) is in metres

**An intuitive way to view eye corrections**

Near-sighted eye is elongated, image of distant object forms in front of retina

Add diverging lens, image forms on retina

Far-sighted eye is short, image of close object forms behind retina

Add converging lens, image forms on retina
DEFECTS OF VISION

Myopia – Short sightedness

Due to enlarged size of the eye ball, the images of distant objects will be focused in front of the retina. This defect is known as Myopia.

Remedy is the use of a concave lens as shown.

Due to the reduced size of the eye ball, the images of nearby objects will be focused behind the retina. This defect is known as Hypermetropia.

Remedy is the use of a convex lens as shown.

This little Piggy

In The Lord of the Flies, Piggy’s glasses are used to focus the Sun’s rays and start a fire. What type of lens do you need for this?

Later in the novel, Piggy’s glasses are broken, and poor Piggy has a hard time seeing because he is nearsighted. What type of lenses were in his glasses?

Remember, do your research if you are going to be an author.
An arrow-shaped object is placed in front of a plane mirror as shown below. The image would look like:

a) 

b) 

c) 

d) 

e) 

An illuminated arrow is placed 2 cm in front of a diverging lens with focal length of -6 cm. The image is:

a) real, inverted, smaller than the object
b) virtual, inverted, larger than the object
c) virtual, upright, larger than the object
d) real, upright, larger than the object
e) virtual, upright, smaller than the object

A diverging lens (has a negative focal length) will always create an upright virtual image in front of the object. Since the image distance is smaller than the object distance, the image will be smaller as well.

An object is placed in front of three different optical devices, two lenses and a mirror, with focal points as shown in the figure. Which will produce real images?

a) I only
b) II only
c) III only
d) I and II
e) II and III

A single concave lens produces only virtual images. An object placed inside the focal length of a convex lens will result in a virtual image. This eliminates I and II. An object outside the focal length of a concave mirror will produce an inverted, real image.
Understanding

A concave mirror with a radius of curvature 1.5 m is used to collect light from a distant source. The distance between the image formed and the mirror is closest to:

a) 0.75 m  
b) 1 m  
c) 1.5 m  
d) 2 m  
e) 3 m

Since the object is distant, then the light rays that approach the mirror are parallel. The focus is \( r/2 \) where \( r \) is the radius of curvature. In this example, \( r=1.5 \text{ m} \), so \( f =0.75 \text{ m} \).

Understanding

A student sets up an optics experiment with a converging lens of focal length 10 cm. He places an illuminated arrow 2 cm high at 15 cm from the lens axis. The size of the image:

a) 0.5 cm  
b) 1 cm  
c) 2 cm  
d) 3.5 cm  
e) 4 cm

Since the magnification is \(-2 \) times. The size of the image is \( 2 \text{ cm} \times 2 \) which is \( 4 \text{ cm} \) and the image is inverted.

Understanding

An object is placed in front of a convex mirror. The location of the image is closest to:

a) A  
b) B  
c) C  
d) D  
e) E

A convex (diverging) mirror will produce an upright, smaller, virtual image.
Understanding

For which of the cases will the image of the arrow be virtual and smaller than the object:

a) I only
b) II only
c) III only
d) I and II
e) I and III

Diverging elements like I and III will always produce smaller virtual images. II will produce a virtual image, but it will be larger. This is basically a magnifying glass.

Free Response Problem

The figure above shows an enlarged portion of the glass wall of a fish tank, currently empty so that air (n=1) is on either side of the glass (n=1.5). The glass is 0.5 cm thick. A ray R is incident on the glass at a 30° angle with the normal as shown.

a) On the left figure, continue the ray, showing qualitatively what happens at the next interface.

b) At what distance above the normal line N will the transmitted ray emerge out of the glass?

c) Determine the incident angle at the second interface that will ensure total internal reflection. Could the initial ray R have its incident angle adjusted to make this happen?

d) Suppose the tank is filled with water (n=1.33) as on the right figure. Show qualitatively what happens at the glass water interface.

The reflected light will also have an angle of reflection of 30°.
We can determine $d$ from the geometry:

$$
\tan(19.5^\circ) = \frac{d}{0.5\text{ cm}}
$$

$$
d = 0.5\text{ cm} \tan(19.5^\circ)
$$

$$
= 0.18 \text{ cm}
$$

**b)** At what distance above the normal line $N$ will the transmitted ray emerge out of the glass?

**c)** Determine the incident angle at the second interface that will ensure total internal reflection. Could the initial ray $R$ have its incident angle adjusted to make this happen?

Since the ray that exits into the air has to exit at 90° to be totally reflected, and that we have the incoming ray parallel to the outgoing ray. We would need to incoming ray have an incident angle of 90°, thus indicating that it would not enter the glass, so we could not make it happen.

**d)** Suppose the tank is filled with water (n=1.33) as on the right figure. Show qualitatively what happens at the glass water interface.
Free Response Problem

A converging lens with focal length 4 cm has an object placed 6 cm in front of it. A diverging lens with focal length −8 cm is placed 28 cm behind the first lens.

a) Determine the position of the image formed by the first lens.

b) Draw a ray diagram needed to display the image from first lens.

c) What is the magnification of the image?

d) Determine the position of the image formed by the second lens.

e) Draw a ray diagram needed to display the image from the second lens.

f) Determine the overall magnification and image orientation of final image.

Therefore the image is at the 12 cm + 8 cm = 20 cm position.

Therefore the image is at the 12 cm + 8 cm = 20 cm position.
Free Response Problem
A converging lens with focal length 4 cm has an object placed 6 cm in front of it. A diverging lens with focal length -8 cm is placed 28 cm behind the first lens.

c) What is the magnification of the image?

\[
m = \frac{v}{u} = \frac{-s_2}{s_1} = \frac{-8}{6} = -1.33\]

Therefore the image is twice as big and is inverted.

Free Response Problem
A converging lens with focal length 4 cm has an object placed 6 cm in front of it. A diverging lens with focal length -8 cm is placed 28 cm behind the first lens.

d) Determine the position of the image formed by the second lens.

\[
s_2 = s_1 \frac{f}{s_1 - f} = 6 \frac{4}{6 - 4} = 12 cm\]

Therefore the virtual image is located at 36 cm - 5.33 cm = 30.67 cm

Free Response Problem
A converging lens with focal length 4 cm has an object placed 6 cm in front of it. A diverging lens with focal length -8 cm is placed 28 cm behind the first lens.

e) Draw a ray diagram needed to display the image from the second lens.

Therefore the final virtual image is inverted.
A converging lens with focal length $4 \text{ cm}$ has an object placed $6 \text{ cm}$ in front of it. A diverging lens with focal length $-8 \text{ cm}$ is placed $28 \text{ cm}$ behind the first lens.

f) Determine the overall magnification and image orientation of final image

$$m = \frac{-s_i}{s_o} = \frac{-5.3 \text{ cm}}{16 \text{ cm}} = -0.332$$

Therefore the image is $2/3$ the original size and is inverted.

Two thin converging lenses of focal lengths $10 \text{ cm}$ and $20 \text{ cm}$ are separated by $20 \text{ cm}$. An object is placed $15 \text{ cm}$ in front of the first lens.

a) Determine the position of the image formed by the first lens.

b) What is the magnification of the image?

c) Draw a ray diagram needed to display the image from first lens.

d) Determine the position of the image formed by the second lens.

e) Determine the overall magnification and image orientation of final image.

f) Draw a ray diagram needed to display the image from the second lens.
**Free Response Problem**

Two thin converging lenses of focal lengths 10 cm and 20 cm are separated by 20 cm. An object is placed 15 cm in front of the first lens.

d) Determine the position of the image formed by the second lens.
e) Determine the overall magnification and image orientation of final image.
f) Draw a ray diagram needed to display the image from the second lens.

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**Free Response Problem**

A converging lens with focal length 25 cm has an object placed 150 cm in front of it. A diverging lens with focal length -15 cm is placed 20 cm behind the first lens.

a) Determine the position of the image formed by the first lens.
b) What is the magnification of the image?
c) Draw a ray diagram needed to display the image from first lens.
d) Determine the position of the image formed by the second lens.
e) Determine the overall magnification and image orientation of final image.
f) Draw a ray diagram needed to display the image from the second lens.
A converging lens with focal length 25 cm has an object placed 150 cm in front of it. A diverging lens with focal length -15 cm is placed 20 cm behind the first lens.

d) Determine the position of the image formed by the second lens.
e) Determine the overall magnification and image orientation of final image.
f) Draw a ray diagram needed to display the image from the second lens.