Error Calculation in Addition and Subtraction

Good

Given : $p = x \pm y$

Error: $\delta p = \delta x + \delta y$

Note: we always **add** the errors when we are adding or subtracting values.

Example:

Given : $A_1 = 540 \pm 10\,g$
$A_2 = 72 \pm 1g$
$A_3 = 940 \pm 20\,g$
$A_4 = 97 \pm 2\,g$

Then Calculate: $M = A_1 - A_2 + A_3 - A_4$

Solution: $M = A_1 - A_2 + A_3 - A_4$
$= (540 - 72 + 940 - 97)\,g$
$= 1311g$

$\delta M = \delta A_1 + \delta A_2 + \delta A_3 + \delta A_4$
$= (10 + 1 + 20 + 2)\,g$
$= 33\,g$

Therefore: $M \pm \delta M = 1311 \pm 33\,g$
$= 1310 \pm 30\,g$

Remember error should be to 1 significant digit.

Best

Given : $p = x \pm y$

Error: $\delta p = \sqrt{(\delta x)^2 + (\delta y)^2}$

Example:

Given : $A_1 = 540 \pm 10\,g$
$A_2 = 72 \pm 1g$
$A_3 = 940 \pm 20\,g$
$A_4 = 97 \pm 2\,g$

Then Calculate: $M = A_1 - A_2 + A_3 - A_4$

Solution: $M = A_1 - A_2 + A_3 - A_4$
$= (540 - 72 + 940 - 97)\,g$
$= 1311g$

$$\delta M = \sqrt{(\delta A_1)^2 + (\delta A_2)^2 + (\delta A_3)^2 + (\delta A_4)^2}$$
$$= \sqrt{(10)^2 + (1)^2 + (20)^2 + (2)^2}\,g$$
$$= \sqrt{505}\,g$$
$$\approx 22.4722\,g$$
$$\approx 20\,g$$

Therefore: $M \pm \delta M = 1310 \pm 20\,g$

This method will give a smaller error then the technique on the left, but the error is more accurate.

Error Calculation in Multiplication and division

Good

Given : $p = \dfrac{x \times y}{z}$

Error: $\delta p = |p| \left[\dfrac{\delta x}{|x|} + \dfrac{\delta y}{|y|} + \dfrac{\delta z}{|z|} \right]$

Example:

Given : $A_1 = 200 \pm 2\,g$

$A_2 = 5.5 \pm 0.1 g$

$A_3 = 10.0 \pm 0.4\,g$

Then Calculate: $M = \dfrac{A_1 \times A_2}{A_3}$

Solution: $M = \dfrac{A_1 \times A_2}{A_3}$

$= \left(\dfrac{200 \times 5.5}{10.0} \right) g$

$= 110\,g$

$$\delta M = M \left[\frac{\delta A_1}{|A_1|} + \frac{\delta A_2}{|A_2|} + \frac{\delta A_3}{|A_3|} \right]$$

$$= 110 \left[\frac{2}{200} + \frac{0.1}{5.5} + \frac{0.4}{10.0} \right] g$$

$$= 110 [0.068181818] g$$

$$= 7.5 g$$

$$\approx 8 g$$

Therefore: $M \pm \delta M = 110 \pm 8\,g$

Best

Given : $p = \dfrac{x \times y}{z}$

Error: $\delta p = |p| \sqrt{\left(\dfrac{\delta x}{x} \right)^2 + \left(\dfrac{\delta y}{y} \right)^2 + \left(\dfrac{\delta z}{z} \right)^2}$

Example:

Given : $A_1 = 200 \pm 2\,g$

$A_2 = 5.5 \pm 0.1 g$

$A_3 = 10.0 \pm 0.4\,g$

Then Calculate: $M = \dfrac{A_1 \times A_2}{A_3}$

Solution: $M = \dfrac{A_1 \times A_2}{A_3}$

$= \left(\dfrac{200 \times 5.5}{10.0} \right) g$

$= 110\,g$

$$\delta M = M \sqrt{\left(\frac{\delta A_1}{A_1} \right)^2 + \left(\frac{\delta A_2}{A_2} \right)^2 + \left(\frac{\delta A_3}{A_3} \right)^2}$$

$$= 110 \sqrt{\left(\frac{2}{200} \right)^2 + \left(\frac{0.1}{5.5} \right)^2 + \left(\frac{0.4}{10.0} \right)^2}\; g$$

$$= 110 [0.4506194] g$$

$$= 4.95\,g$$

$$\approx 5 g$$

Therefore: $M \pm \delta M = 110 \pm 5\,g$

Error Calculations in Other Functions

The safest way to calculate errors when you encounter functions that contain operations other than addition, subtraction, multiplication or division is to calculate the value of the function, then calculate the largest and the smallest probable value of the function by including the errors. The maximum difference will provide you the error.

Example:

Let $g = 9.81\frac{m}{s^2}$

$d = 0.204 \pm 0.004\,m$

$\theta = 66° \pm 3°$

Calculate $v = \sqrt{\frac{dg}{\sin(2\theta)}}$

Solution:

First determine the value of v

$$v = \sqrt{\frac{dg}{\sin(2\theta)}} = \sqrt{\frac{(0.204)(9.81)}{\sin(2\times 66°)}}\,\frac{m}{s} \approx 1.641016\frac{m}{s}$$

The largest value will occur when the numerator is large and the denominator is small
The smallest value will occur when the numerator is small and the denominator is arge

Large

$$v = \sqrt{\frac{dg}{\sin(2\theta)}} = \sqrt{\frac{(0.204+0.004)(9.81)}{\sin(2\times(66+3)°)}}\,\frac{m}{s} \approx 1.746267\,m$$

Small

$$v = \sqrt{\frac{dg}{\sin(2\theta)}} = \sqrt{\frac{(0.204-0.004)(9.81)}{\sin(2\times(66-3)°)}}\,\frac{m}{s} \approx 1.55728\,m$$

Let's determine the **largest** difference:

1.746267-1.641016 =**0.105251**
1.641016-1.55728=0.083736

Therefore we have $v = 1.6 \pm 0.1\frac{m}{s}$