**Waves**

Ensure Sound is on.

**Reflection**

\[ \theta = \theta \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

**Light Doesn’t Just Bounce**

***It Also Refracts!***

Reflected: Bounces  *(Mirrors!)*

\[ \theta_1 = \theta_2 \]

\[ \frac{1}{d_e} + \frac{1}{d_i} = \frac{1}{f} \]

Refracted: Bends  *(Lenses!)*

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

**Index of Refraction**

300,000,000 m/second: it’s not just a good idea, it’s the law!

\[ v = \frac{c}{n} \]

Speed of light in medium

\[ v < c \]  

**SO**  \( n > 1 \) always!
When light travels from one medium to another the speed changes $v = \frac{c}{n}$, but the frequency is constant. So the light bends:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Preflight

1. $n_1 > n_2$
2. $n_1 = n_2$
3. $n_1 < n_2$

$\theta_1 < \theta_2$

$\sin(\theta_1) < \sin(\theta_2)$

Preflight

$\theta_1 = \sin^{-1}(n_2/n_1)$ then $\theta_2 = 90$°

Light incident at a larger angle will only have reflection ($\theta_1 = \theta_2$)

Total Internal Reflection

Recall Snell's Law: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

$n_1 > n_2 \Rightarrow \theta_2 > \theta_1$

Snell's Law Practice

Usually, there is both reflection and refraction!

A ray of light traveling through the air ($n=1$) is incident on water ($n=1.33$). Part of the beam is reflected at an angle $\theta_1 = 60$. The other part of the beam is refracted. What is $\theta_2$?

$\theta_1 = 60^\circ$

$\sin(60^\circ) = 1.33 \sin(\theta_2)$

$\theta_2 = 40.6$°

Review Equations (1)

$c = \frac{\lambda f}{n}$

$n = \frac{c}{v}$

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

$n_1 > n_2$
Light travels from air into water. If it enters water at 60° to the surface, find the angle at which it is refracted and the speed at which it moves in the water ($n_{\text{water}} = 1.33$).

\[
\sin(\theta_1) = \frac{n_1}{n_2} \sin(\theta_2)
\]

\[
\theta_1 = \sin^{-1} \left( \frac{n_1}{n_2} \sin(\theta_2) \right)
\]

\[
\theta_2 = \sin^{-1} \left( \frac{1.00}{1.33} \sin(30°) \right) \approx 22°
\]

The critical angle for an air-water boundary ($n_{\text{water}} = 1.33, n_{\text{air}} = 1.00$) is

\[
\sin(\theta) = \frac{n_2}{n_1} \quad n_1 > n_2
\]

\[
\theta = \sin^{-1} \left( \frac{n_2}{n_1} \right)
\]

\[
\theta = \sin^{-1} \left( \frac{1.00}{1.33} \right) \approx 49°
\]

A wave moves from medium 1 to medium 2, given that $\theta_2 = 30°$, $\lambda_2 = 0.50$ cm, $v_2 = 3.0$ cm/s, and the index of refraction for $n_2 = 1.3$ and $n_1 = 1.0$, find $v_1$, $\lambda_1$, and $d$.

\[
d = \frac{d}{n_1}
\]

Apparent Depth

\[
d' = d \frac{n_2}{n_1}
\]
Can the person standing on the edge of the pool be prevented from seeing the light by total internal reflection?

1) Yes  2) No

"There are millions of light 'rays' coming from the light. Some of the rays will be totally reflected back into the water, but most of them will not."

Understanding Refraction

As we pour more water into bucket, what will happen to the number of people who can see the ball?

1) Increase  2) Same  3) Decrease

Understanding Refraction

As we pour more water into bucket, what will happen to the number of people who can see the ball?

1) Increase  2) Same  3) Decrease

Fiber Optics

At each contact w/ the glass air interface, if the light hits at greater than the critical angle, it undergoes total internal reflection and stays in the fiber.

Total Internal Reflection only works if \( n_{\text{outside}} < n_{\text{inside}} \)
Fiber Optics

At each contact with the glass air interface, if the light hits at greater than the critical angle, it undergoes total internal reflection and stays in the fiber.

Add "cladding" so outside material doesn’t matter!

We can be certain that $n_{\text{cladding}} < n_{\text{inside}}$

Polarization

- Transverse waves have a polarization
  - (Direction of oscillation of E field for light)

Types of Polarization

- Linear (Direction of E is constant)
- Circular (Direction of E rotates with time)
- Unpolarized (Direction of E changes randomly)

Linear Polarizers

Linear Polarizers absorb all electric fields perpendicular to their transmission axis.

Dichroic (Polaroid) materials have the property of selectively developing one of two orthogonal components of ordinary light. The crystal actual align opposite to what you would intuitively expect waves to be blocked. i.e. vertical alignment blocks vertical electric fields, by absorbing their energy and oscillating the electrons in a vertical manner. Horizontal waves will pass through.

Reflected Light is Polarized

Reflected light is polarized.
Unpolarized Light on Linear Polarizer

- Most light comes from electrons accelerating in random directions and is unpolarized.
- Averaging over all directions:

\[ S_{\text{transmitted}} = \frac{1}{2} S_{\text{incident}} \]

Always true for unpolarized light!

Linearly Polarized Light on Linear Polarizer (Law of Malus)

\[ E_{\text{transmitted}} = E_{\text{incident}} \cos(\theta) \]

\[ S_{\text{transmitted}} = S_{\text{incident}} \cos^2(\theta) \]

\[ \theta \] is the angle between the incoming light's polarization, and the transmission axis.

Intensity

\[ S \propto E^2 \]

\[ S_{\text{max}} \propto \left[ E_0 \cos(\theta) \right]^2 \]

\[ = E_0^2 \cos^2(\theta) \]

\[ = E_0^2 \left( \frac{1}{2} \right) \]

\[ = \frac{1}{2} E_0^2 \]

\[ = \frac{1}{2} S_0 \]

The average of \( \cos^2(\theta) \)

\[ \frac{1}{2} \int_0^{2\pi} \cos^2(\theta) d\theta = \frac{1}{2} \int_0^{2\pi} \left( \frac{1}{2} + \cos(2\theta) \right) d\theta \]

\[ = \frac{1}{2} \left[ \theta + \frac{1}{2} \sin(2\theta) \right] \bigg|_0^{2\pi} \]

\[ = \pi \]

\[ \frac{1}{2} S \]

Question

Unpolarized light (like the light from the sun) passes through a polarizing sunglass (a linear polarizer). The intensity of the light when it emerges is

1. zero
2. \( \frac{1}{2} \) what it was before
3. \( \frac{1}{4} \) what it was before
4. \( \frac{1}{3} \) what it was before
5. Need more information

Now, horizontally polarized light passes through the same glasses (which are vertically polarized). The intensity of the light when it emerges is

1. zero
2. \( \frac{1}{2} \) what it was before
3. \( \frac{1}{4} \) what it was before
4. \( \frac{1}{3} \) what it was before
5. Need more information
**Answer for 1**

Unpolarized light (like the light from the sun) passes through a polarizing sunglasses (a linear polarizer). The intensity of the light when it emerges is

1. zero
2. 1/2 what it was before
3. 1/4 what it was before
4. 1/3 what it was before
5. need more information

\[ S_{\text{max}} = \frac{1}{2} S_0 \]

**Answer for 2**

Now, horizontally polarized light passes through the same glasses (which are vertically polarized). The intensity of the light when it emerges is

- zero
- 1/2 what it was before
- 1/4 what it was before
- 1/3 what it was before
- need more information

Since all light hitting the sunglasses was horizontally polarized and only vertical rays can get thru, the glasses block everything.

---

**Law of Malus – 2 Polarizers**

1) Intensity of unpolarized light incident on linear polarizer is reduced by \( \frac{1}{2} \). \( S_1 = \frac{1}{2} S_2 \)

2) Light transmitted through first polarizer is vertically polarized. Angle between it and second polarizer is \( \theta = 90^\circ \). \( S_2 = S_1 \cos(90^\circ) = 0 \)

**Cool Link**

---

**Law of Malus – 3 Polarizers**

1) Light will be vertically polarized with: \( \theta = 45^\circ \)

2) Light transmitted through first polarizer is vertically polarized. Angle between it and second polarizer is \( \theta = 45^\circ \). \( I_1 = I_0 \cos^2(45^\circ) = \frac{1}{2} I_0 \cos^2(45^\circ) \)

3) Light transmitted through second polarizer is polarized \( 45^\circ \) from vertical. Angle between it and third polarizer is \( \theta = 45^\circ \). \( I_3 = I_1 \cos^2(45^\circ) = \frac{1}{2} I_1 \cos^2(45^\circ) \)

Angle is 45 degrees with respect to last TA.

Remember you can use I or S for intensity.
Light

The incident light can have the electric field of the light waves lying in the same plane as the that it is traveling. Light with this polarization is said to be \( p \)-polarized, because it is parallel to the plane. Light with the perpendicular polarization is said to be \( s \)-polarized, from the German *senkrecht*—perpendicular.

Brewster’s angle

Reflected light is partially polarized (more horizontal than vertical). But...

\[
\tan \theta_B = \frac{n_2}{n_1}
\]

\( \theta_B \) = Brewster’s angle

...when angle between reflected beam and refracted beam is exactly 90 degrees, reflected beam is 100% horizontally polarized!

\[
\frac{n_1 \sin \theta_B}{n_2} = n_2 \sin (90-\theta_B)
\]

\[
n_1 \sin \theta_B = n_2 \cos \theta_B
\]

ACT: Law of Malus

\[
S_0 = S_2 \cos^2(60)
\]

\[
S_1 = S_2 \cos^2(30)
\]

\[
S_2 = S_0 \cos^2(60) - S_0 \cos^2(30)
\]

1) \( S_0 > S_2 \)

2) \( S_0 = S_2 \)

3) \( S_0 < S_2 \)
Polarized so that the oscillation is in same direction as to be flat with reflecting surface (polarized in the plane of reflecting surface).

Brewster’s Angle

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

Since \( \theta_1 + \theta_2 = 90^\circ \) then

\[ \sin(\theta_1) = \cos(\theta_2) \]

Polarizing sunglasses are often considered to be better than tinted glasses because they...
- block more light
- block more glare
- are safer for your eyes
- are cheaper

Polarizing sunglasses (when worn by someone standing up) work by absorbing light polarized in which direction?
- horizontal
- vertical

Preflight

When glare is around \( \theta_B \), it’s mostly horiz. polarized!

Brewster’s Angle from air off glass

\[ \tan(\theta) = \frac{n_{glass}}{n_{air}} \]

\[ \tan(\theta) = \frac{1.5}{1.0} \]

\[ \theta \approx 56^\circ \]
How Sunglasses Work

Any light ray that comes from a reflection will be slightly horizontally polarized. The glasses will only allow vertically (or vertical component of unpolarized light) to pass through to the eye. Thus reducing the glare.

ACT: Brewster’s Angle

When a polarizer is placed between the light source and the surface with transmission axis aligned as shown, the intensity of the reflected light:

1. Increases
2. Unchanged
3. Decreases

T.A.

Dispersion

The index of refraction n depends on color!

In glass: \( n_{\text{blue}} = 1.53 \)  \( n_{\text{red}} = 1.52 \)

\( n_{\text{blue}} > n_{\text{red}} \)

Blue light gets deflected more

Understanding

Skier sees blue coming up from the bottom (1), and red coming down from the top (2) of the rainbow.
LIKE SO!

In second rainbow pattern is reversed

Interference Requirements

- Need two (or more) waves
- Must have same frequency
- Must be coherent (i.e. waves must have definite phase relation)

Interference for Sound ...

For example, a pair of speakers, driven in phase, producing a tone of a single \( f \) and \( \lambda \):

hmmm... I'm just far enough away that \( l_2 - l_1 = \lambda/2 \), and I hear no sound at all!

But this won't work for light rays--can't get coherent pair of sources

Interference for Light ...

- Can't produce coherent light from separate sources. (\( f = 10^{14} \) Hz)
- Need two waves from single source taking two different paths
  - Two slits
  - Reflection (thin films)
  - Diffraction
Understanding

The experiment is modified so that one of the waves has its phase shifted by $\frac{1}{2} \lambda$. Now, the interference will be:

1) Constructive
2) Destructive
3) Depends on $L$

Young’s Double Slit Concept

At points where the difference in path length is $0, \lambda, 2\lambda, \ldots$, the screen is bright. (constructive)

At points where the difference in path length is $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$, the screen is dark. (destructive)

Young’s Double Slit Key Idea

Two rays travel almost exactly the same distance. (Screen must be very far away: $L \gg d$)

Bottom ray travels a little further.

Key for interference is this small extra distance.
Young's Double Slit Quantitative

\[ \text{Path length difference} = d \sin(\theta) \]

Constructive interference
(Where \( m = 0, 1, 2, \ldots \))

\[ d \sin(\theta) = m\lambda \]

Destructive interference
(Where \( m = 0, 1, 2, \ldots \))

\[ d \sin(\theta) = (m + \frac{1}{2})\lambda \]

Equations

1. \([PS_1 - PS_2] = m\lambda, \quad m = 0, 1, 2\]
2. \([PS_1 - PS_2] = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2\]
3. \(d \sin(\theta) = m\lambda\)
4. \(d \sin(\theta) = (m + \frac{1}{2})\lambda\)
5. \(\frac{y}{L} = \frac{m\lambda L}{d}\)
6. \(\frac{y}{L} = \frac{(m + \frac{1}{2})\lambda L}{d}\)

If You only know the Distances between Sources and Point (or Large Distances)

Constructive interference
\([PS_1 - PS_2] = m\lambda, \quad m = 0, 1, 2\]

Destructive interference
\([PS_1 - PS_2] = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2\]

Understanding

When this Young's double slit experiment is placed under water. The separation \( y \) between minima and maxima...

1) increases
2) same
3) decreases

wavelength is shorter under water \( \lambda_2 = \frac{n_1}{n_2} \lambda_1 \)
Understanding

In the Young double slit experiment, is it possible to see interference maxima when the distance between slits is smaller than the wavelength of light?

1) Yes  2) No

Need:
\[ d \sin \theta = m \lambda \]

If \( \lambda < d \) then \( \lambda / d < 1 \)

so \( \sin \theta > 1 \)

Not possible!

Understanding

If one source to the screen is \( 1.2 \times 10^{-5} \) m farther than the other source and red light with wavelength 600 nm is used, determine the order number of the bright spot.

Path length difference = \( |P_s - P_S| = 1.2 \times 10^{-5} \) m

We don’t know the angle, so we use

\[ m = \frac{|P_s - P_S| - 4.50 \times 10^{-7}}{6.00 \times 10^{-7} \text{ m}} = 20 \]

Understanding

Light of wavelength 450 nm shines through openings 3.0 um apart. At what angle will the first-order maximum occur? What is the angle of the first order minimum?

\[ m = \frac{P_s - P_S}{d} = \frac{1}{2} \]

\[ d \sin \theta = m \lambda \]

\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) \]

\[ = \sin^{-1} \left( \frac{1}{3.0 \times 10^{-6} \text{ m}} \right) \]

\[ = 8.6^\circ \]

First order max starts at \( m = 1 \),

first order min starts at \( m = 0 \)

Understanding

A point P is on the second-order maximum, 65 mm from one source and 45 mm from another source. The sources are 40 mm apart. Find the wavelength of the wave and the angle the point makes in the double slit.

\[ |P_s - P_S| = 65 \text{ mm} - 45 \text{ mm} = 20 \text{ mm} \]

\[ \frac{12 \text{ mm}}{40 \text{ mm}} = 0.3 \]

\[ d \sin \theta = m \lambda \]

\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) \]

\[ = \sin^{-1} \left( \frac{0.3}{3.0 \times 10^{-6} \text{ m}} \right) \]

\[ = 4.3^\circ \]
Understanding

For light of wavelength 650 nm, what is the spacing of the bright bands on a screen 1.5 m away if the distance between slits is 6.6x10^{-6} m?

Since we want the spacing on the bands and are given the distance the screen is from the slits,

\[ y = \frac{m\lambda}{d} \]

\[ y = \frac{m(6.5) \times 10^{-7}}{6.6 \times 10^{-6}} (1.5) \]

\[ y = 0.15 \text{ m} \]

Therefore the distance between the principle bright band and the first order band is 0.15 m (this is actually the distance between any two bright bands).

Multiple Slits

(Diffraction Grating – N slits with spacing d)

Path length difference 1-2 = d \sin \theta = \lambda
Path length difference 1-3 = 2d \sin \theta = 2\lambda
Path length difference 1-4 = 3d \sin \theta = 3\lambda

Constructive interference for all paths when

\[ d \sin \theta = m \lambda \]

Understanding

All 3 rays are interfering constructively at the point shown. If the intensity from ray 1 is I_0, what is the combined intensity of all 3 rays?

1) I_0
2) 3I_0
3) 9I_0

When rays 1 and 2 are interfering destructively, is the intensity from the three rays a minimum?

1) Yes
2) No

Three slit interference

\[ 9I_0 \]

\[ \frac{2\lambda}{3} \]

\[ \frac{\lambda}{3} \]

\[ \frac{\lambda}{2} \]

\[ \frac{2\lambda}{3} \]
### Multiple Slit Interference (Diffraction Grating)

For many slits, maxima are still at
\[
\sin(\theta) = m \frac{\lambda}{d}
\]
Region between maxima gets suppressed more and more as no. of slits increases – bright fringes become narrower and brighter.

2 slits (N=2)  10 slits (N=10)

<table>
<thead>
<tr>
<th># of Slits</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

### Understanding

Compare the spacing produced by a diffraction grating with 5500 lines in 1 cm for red light (620 nm) and green light (510 nm). The distance to the screen is 1.1 m.

First we need to determine d the distance between the grates

\[
d = \frac{1.1 \times 10^{-5} \text{ m}}{5500 \text{ lines}} = 2.0 \times 10^{-8} \text{ m}
\]

Note: the larger spacing for red than for green.

Now for the spacing, y

\[
y = \frac{L \lambda}{d}
\]

\[
y_{\text{red}} = \frac{(1.1 \text{ m})(6.20 \times 10^{-7} \text{ m})}{2.0 \times 10^{-8} \text{ m}} = 0.34 \text{ m}
\]

\[
y_{\text{green}} = \frac{(1.1 \text{ m})(5.10 \times 10^{-7} \text{ m})}{2.0 \times 10^{-8} \text{ m}} = 0.28 \text{ m}
\]

### Understanding

A diffraction grating has 1000 slits/cm. When light of wavelength 480 nm is shone through the grating, what is the separation of the bright bands 4.0 m away? How many orders of this light are seen?

First we need to determine d the distance between the grates

\[
d = \frac{0.01 \text{ m}}{1000 \text{ lines}} = 1.0 \times 10^{-7} \text{ m}
\]

Now for the spacing, y

\[
y = \frac{L \lambda}{d}
\]

\[
y_{\text{red}} = \frac{(4.0 \text{ m})(4.8 \times 10^{-7} \text{ m})}{1.0 \times 10^{-7} \text{ m}} = 0.192 \text{ m}
\]

For max we will set the angle \( \theta \) to 90°

\[
d \sin(\theta) = m \lambda
\]

\[
\frac{d}{2} = \frac{d}{2} \sin(90°) = 4.0 \times 10^{-5} \text{ m}
\]

\[
4.8 \times 10^{-7} \text{ m}
\]

\[
= 20.8
\]
Single Slit Interference?!

In this theory, every point on a wave front is considered a point source for tiny secondary wavelets, spreading out in front of the wave as the same speed of the wave itself.

This is not what is actually seen!

Diffraction Rays

Huygens’ Wave Model

Every point on a wave front acts as a source of tiny wavelets that move forward.

Light waves originating at different points within opening travel different distances to wall, and can interfere!

We will see maxima and minima on the wall.
**Huygens' Principle**

The Dutch scientist Christian Huygens, a contemporary of Newton, proposed *Huygens' Principle*, a geometrical way of understanding the behavior of light waves.

**Huygens Principle:** Consider a wave front of light:

1. Each point on the wave front is a new source of a spherical wavelet that spreads out spherically at wave speed.
2. At some later time, the new wave front is the surface that is tangent to all of the wavelets.

**Single Slit Diffraction**

As we have seen in the demonstrations with light and water waves, when light goes through a narrow slit, it spreads out to form a diffraction pattern.

Now, we want to understand this behavior in more detail.

Under this condition, every ray originating in top half of slit interferes destructively with the corresponding ray originating in bottom half.

\[ \sin \theta = \frac{\lambda}{W} \Rightarrow \theta = \frac{\lambda}{W} \]

\[ 1^\text{st} \text{minima at:} \quad \sin \theta = \frac{1}{2} \]
### Conditions for Diffraction Minima

The conditions for diffraction minima are given by:

\[ \sin \theta_m = \frac{m \lambda}{w} \]

where:
- \( \theta_m \) is the angle of the mth dark fringe
- \( m \) is an integer
- \( \lambda \) is the wavelength of the light
- \( w \) is the width of the slit

### Analyzing Single Slit Diffraction

For an open slit of width \( w \), subdivide the opening into segments and imagine a Huygen wavelet originating from the center of each segment. The wavelets going forward (\( \theta = 0 \)) all travel the same distance to the screen and interfere constructively to produce the central maximum.

Now consider the wavelets going at an angle such that \( \lambda = w \sin \theta = \theta \). The wavelet pair \((1, 2)\) has a path length difference \( D_{12} = \frac{\lambda}{2} \) and therefore will cancel. The same is true of wavelet pairs \((3,4), (5,6), \ldots\). Moreover, if the aperture is divided into \( m \) subparts, this procedure can be applied to each subpart. This procedure locates all of the dark fringes.

### Single Slit Diffraction Summary

- **Condition for quarters of slit to destructively interfere**
  \[ \frac{w}{4} \sin \theta = \frac{\lambda}{2} \]

- **Condition for halves of slit to destructively interfere**
  \[ w \sin \theta = \lambda \]

- **Condition for sixths of slit to destructively interfere**
  \[ \frac{w}{6} \sin 3\theta = \lambda \]

**Note:** Interference only occurs when the width \( w > \lambda \).
Understanding

A slit is $2.0 \times 10^{-5} \text{ m}$ wide and is illuminated by light of wavelength 550 nm. Determine the width of the central maximum, in degrees and in centimetres when $L=0.30\text{ m}$?

We will find the position of the first order minimum, which is one-half the total width of the central maximum.

$$m\lambda = w\sin(\theta)$$

$$m\lambda = \frac{\lambda y}{L}$$

$$\sin(\theta) = \frac{m\lambda}{w}$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{w}\right)$$

$$= \sin^{-1}\left(\frac{(5.50 \times 10^{-7} \text{ m})}{(2.0 \times 10^{-5} \text{ m})}\right)$$

$$= 1.6^\circ$$

The full width is twice this, $3.20 \text{ m}$.

$$y_n = \frac{m\lambda L}{w}$$

$$= \left(1\right)\left(5.50 \times 10^{-7} \text{ m}\right)\left(0.30\text{ m}\right)$$

$$= 0.00825 \text{ m}$$

Therefore the width of Central Maximum is $2(0.00825\text{ cm}) = 1.65 \text{ cm}$.

Example: Diffraction of a laser through a slit

Light from a helium-neon laser ($\lambda = 633 \text{ nm}$) passes through a narrow slit and is seen on a screen 2.0 m behind the slit. The first minimum of the diffraction pattern is observed to be located 1.2 cm from the central maximum.

How wide is the slit?

$$w = \frac{L\lambda}{y_1} = \frac{(2.00\text{ m})(6.33 \times 10^{-7} \text{ m})}{(1.2 \times 10^{-2} \text{ m})} = 1.06 \times 10^{-4} \text{ m}$$

$$\theta = \frac{y_1}{L} = \frac{(0.012 \text{ m})}{(2.00\text{ m})} = 0.0060 \text{ rad}$$

$$w = \frac{\lambda}{\sin\theta} = \frac{6.33 \times 10^{-7} \text{ m}}{(6.00 \times 10^{-4} \text{ rad})} = 1.06 \times 10^{-4} \text{ m} = 0.106 \text{ mm}$$

Understanding

Light is beamed through a single slit 1.0 mm wide. A diffraction pattern is observed on a screen 2.0 m away. If the central band has a width of 2.5 mm, determine the wavelength of the light.

To use the equations, we need half of the width.

$$m\lambda = \frac{\lambda y}{L}$$

$$\lambda = \frac{w}{L}$$

$$= \frac{(1.0 \times 10^{-3} \text{ m})(1.25 \times 10^{-7} \text{ m})}{(2.00\text{ m})(1)}$$

$$= 6.25 \times 10^{-10} \text{ m}$$

Width of a Single-Slit Diffraction Pattern

$$y_n = \frac{m\lambda L}{w}$$

$m = 1, 2, 3, \ldots$ (positions of dark fringes)

Width $= \frac{2L}{w}$ (width of diffraction peak from min to min)
Two single slit diffraction patterns are shown. The distance from the slit to the screen is the same in both cases. Which of the following could be true?

(a) The slit width \( w \) is the same for both; \( \lambda_1 > \lambda_2 \).
(b) The slit width \( w \) is the same for both; \( \lambda_1 < \lambda_2 \).
(c) The wavelength is the same for both; width \( w_1 < w_2 \).
(d) The slit width and wavelength is the same for both; \( m_1 < m_2 \).
(e) The slit width and wavelength is the same for both; \( m_1 > m_2 \).

\[
\sin(\theta) = \frac{m \lambda}{w} \quad (m=1, 2, 3, \ldots)
\]

What is the wavelength of a ray of light whose first side diffraction maximum is at 15°, given that the width of the single is 2511 nm?

\[
\lambda = \frac{\sin(\theta) w}{m}
\]

We want the first maximum, therefore \( m \) is approximately 1.5.

\[
\lambda = \frac{(2511 \text{ nm}) \sin(15°)}{1.5}
\]

\[
\lambda = 430 \text{ nm}
\]

Combined Diffraction and Interference (FYI only)

So far, we have treated diffraction and interference independently. However, in a two-slit system both phenomena should be present together.

\[
i_{\text{diff}} = 4I_{\text{ difficult}} \sin^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)
\]

\[
d = \frac{\pi \lambda}{\Lambda}, \quad \Lambda = \frac{\pi d}{\lambda}
\]

Notice that when \( d \) is an integer, diffraction minima will fall on top of “missing” interference maxima.

Diffraction from Circular Aperture

Maxima and minima will be a series of bright and dark rings on screen.

First diffraction minimum is at \( \sin \theta = 1.22 \frac{\lambda}{D} \).
Understanding

A laser is shone at a screen through a very small hole. If you make the hole even smaller, the spot on the screen will get:

\[
\sin \theta = \frac{\lambda}{D} \quad (1) \text{Larger} \quad (2) \text{Smaller}
\]

Which drawing correctly depicts the pattern of light on the screen?

(1) (2) (3) (4)

\[
\sin \theta \approx \sin \frac{\lambda}{D}
\]

Intensity from Circular Aperture

First diffraction minima

\[
\sin \theta = \frac{\lambda}{D}
\]

Resolving Power

To see two objects distinctly, need \( \theta_{\text{objects}} > \theta_{\text{min}} \)

\( \theta_{\text{objects}} \) is angle between objects and aperture:

\[
\tan \theta_{\text{objects}} = \frac{d}{y}
\]

\( \theta_{\text{min}} \) is minimum angular separation that aperture can resolve:

\[
\sin \theta_{\text{min}} = \frac{1.22 \lambda}{D}
\]

Two objects are just resolved when the maximum of one is at the minimum of the other.

These objects are just resolved

Improve resolution by increasing \( \theta_{\text{objects}} \) or decreasing \( \theta_{\text{min}} \).
Understanding

Astronaut Joe is standing on a distant planet with binary suns. He wants to see them but knows it’s dangerous to look straight at them. So he decides to build a pinhole camera by poking a hole in a card. Light from both suns shines through the hole onto a second card.

But when the camera is built, Astronaut Joe can only see one spot on the second card! To see the two suns clearly, should he make the pinhole larger or smaller?

\[ \theta_{\text{min}} = 1.22\frac{\lambda}{D} \]

ACT: Resolving Power

How does the maximum resolving power of your eye change when the brightness of the room is decreased.

1) Increases
2) Constant
3) Decreases

When the light is low, your pupil dilates (D can increase by factor of 10). But actual limitation is due to density of rods and cones, so you don’t notice an much of an effect!

Summary

- **Interference:** Coherent waves
  - Full wavelength difference = Constructive
  - \( \frac{1}{2} \) wavelength difference = Destructive
- **Multiple Slits**
  - Constructive: \( d \sin(\theta) = m \lambda \) \( (m=1,2,3\ldots) \)
  - Destructive: \( d \sin(\theta) = (m + \frac{1}{2}) \lambda \) \( 2 \) slit only
  - More slits = brighter max, darker mins
- **Huygens’ Principle:** Each point on wave front acts as coherent source and can interfere.
- **Single Slit:**
  - Destructive: \( w \sin(\theta) = m \lambda \) \( (m=1,2,3\ldots) \)

Thin Film Interference

Get two waves by reflection off of two different interfaces.

Ray 2 travels approximately \( 2t \) further than ray 1.
Reflection + Phase Shifts

Upon reflection from a boundary between two transparent materials, the phase of the reflected light may change.

- If \( n_1 > n_2 \) - no phase change upon reflection.
- If \( n_1 < n_2 \) - phase change of 180° upon reflection. (equivalent to the wave shifting by \( \lambda/2 \)).

Thin Film Summary (1)

Determine \( \delta \), number of extra wavelengths for each ray.

- Ray 1: \( \delta_1 = 0 \) or \( \frac{1}{2} \)
- Ray 2: \( \delta_2 = 0 \) or \( \frac{1}{2} + 2t/\lambda_{\text{film}} \)

Note: this is wavelength in film!

If |(\( \delta_2 - \delta_1 \))| = 0, 1, 2, 3 .... (m) constructive
If |(\( \delta_2 - \delta_1 \))| = \( \frac{1}{2} \), 1 \( \frac{1}{2} \), 2 \( \frac{1}{2} \) .... (m + \( \frac{1}{2} \)) destructive

Thin Film Summary (2)

- Phase shift after reflection
- No phase shift after reflection
- Phase shift

Thin Film Practice

Blue light (\( \lambda_0 = 500 \text{ nm} \)) incident on a glass (\( n_{\text{glass}} = 1.5 \)) cover slip (\( t = 167 \text{ nm} \)) floating on top of water (\( n_{\text{water}} = 1.3 \)).

Is the interference constructive or destructive or neither?

\( \delta_1 = \frac{1}{2} \) Reflection at air-film interface only
\( \delta_2 = 0 + 2t/\lambda_{\text{glass}} = 2t n_{\text{glass}}/\lambda_0 = 1 \)
Phase shift = \( \delta_2 - \delta_1 = \frac{1}{2} \) wavelength
**ACT: Thin Film**

Blue light \( \lambda = 500 \text{ nm} \) is incident on a very thin film (\( t = 167 \text{ nm} \)) of glass on top of plastic. The interference is:

1. **constructive**
2. **destructive**
3. **neither**

\[
\begin{align*}
\delta_1 &= \frac{\lambda}{2} \\
\delta_2 &= \frac{\lambda}{2} + 2t / n_{\text{glass}} \\
\text{Phase shift} &= \delta_2 - \delta_1 = 1 \text{ wavelength}
\end{align*}
\]

**Understanding**

A thin film of gasoline \((n_{\text{gas}}=1.20)\) and a thin film of oil \((n_{\text{oil}}=1.45)\) are floating on water \((n_{\text{water}}=1.33)\). When the thickness of the two films is exactly one wavelength…

\[
\begin{align*}
\delta_{1,\text{gas}} &= \frac{\lambda}{2} \\
\delta_{2,\text{gas}} &= \frac{\lambda}{2} + 2 \\
\delta_{1,oil} &= \frac{\lambda}{2} \\
\delta_{2,oil} &= \frac{\lambda}{2} + 2t / n_{\text{oil}} \\
|\delta_{2,\text{gas}} - \delta_{1,\text{gas}}| &= 2 \\
|\delta_{2,oil} - \delta_{1,oil}| &= 3/2
\end{align*}
\]

**Understanding**

Light travels from air to a film with a refractive index of 1.40 to water \((n=1.33)\). If the film is 1020 nm thick and the wavelength of light is 476 nm in air, will constructive or destructive interference occur?

Ray 1: Since \( n_{\text{air}} < n_{\text{film}} \), we have a phase shift of \( \lambda/2 \) when it reflects of the film.

Ray 2: Since \( n_{\text{film}} > n_{\text{water}} \), we have no phase shift. So we need only determine the number of extra wavelengths travelled by the ray.

\[
\begin{align*}
\delta_1 &= 0 + \frac{2t}{\lambda_{\text{film}}} \\
\delta_2 &= 0 + \frac{2t}{\lambda_{\text{water}}} \\
\text{Phase shift} &= |\delta_2 - \delta_1| = 5\frac{1}{2} \text{ wavelength}
\end{align*}
\]

**Understanding**

A camera lens \((n=1.50)\) is coated with a film of magnesium fluoride \((n=1.25)\). What is the minimum thickness of the film required to minimize reflected light of wavelength 550 nm?

Ray 1: Since \( n_{\text{air}} < n_{\text{film}} \), we have a phase shift of \( \lambda/2 \) when it reflects of the film.

Ray 2: Since \( n_{\text{film}} > n_{\text{lens}} \), we have a phase shift of \( \lambda/2 \) when it reflects of the lens.

\[
\begin{align*}
\delta_1 &= \frac{\lambda}{2} \\
\delta_2 &= \frac{\lambda}{2} + 2t / n_{\text{film}} \\
\delta_{\text{film}} &= \frac{\lambda_{\text{film}}}{2} \\
\text{Phase shift} &= |\delta_2 - \delta_1| = \frac{3}{2}
\end{align*}
\]
What do we see?
- Our eyes can't detect intrinsic light from objects (mostly infrared), unless they get “red hot”
- The light we see is from the sun or from artificial light (bulbs, etc.)
- When we see objects, we see reflected light
  - immediate bouncing of incident light (zero delay)
- Very occasionally we see light that has been absorbed, then re-emitted at a different wavelength
  - called fluorescence, phosphorescence, luminescence.

White light
- White light is the combination of all wavelengths, with equal representation
  - “red hot” poker has much more red than blue light
  - experiment: red, green, and blue light bulbs make white
- RGB monitor combines these colors to display white

Colours
- Light is characterized by frequency, or more commonly, by wavelength
- Visible light spans from 400 nm to 700 nm
  - or 0.4 μm to 0.7 μm; 0.0004 mm to 0.0007 mm, etc.
- White light is the combination of all wavelengths, with equal representation
  - “red hot” poker has much more red than blue light
  - experiment: red, green, and blue light bulbs make white
  - RGB monitor combines these colors to display white
**Introduction to Spectra**

- We can make a spectrum out of light, dissecting its constituent colors
  - A prism is one way to do this
  - A diffraction grating also does the job
- The spectrum represents the wavelength-by-wavelength content of light
  - can represent this in a color graphic like that above
  - or can plot intensity vs. wavelength

**Why do things look the way they do?**

- Why are metals shiny?
  - Recall that electromagnetic waves are generated from accelerating charges (i.e., electrons)
  - Electrons are free to roam in conductors (metals)
  - An EM wave incident on metal readily vibrates electrons on the surface, which subsequently generates EM radiation of exactly the same frequency (wavelength)
  - This indiscriminate vibration leads to near perfect reflection, and exact cancellation of the EM field in the interior of the metal—only surface electrons participate

**What about glass?**

- Why is glass clear?
  - The clear piece of glass is transparent to visible light because the available electrons in the material which could absorb the visible photons have no available energy levels above them in the range of the quantum energies of visible photons. The glass atoms do have vibrational energy modes which can absorb infrared photons, so the glass is not transparent in the infrared. This leads to the greenhouse effect.